Analysis of the X-Junction Between Two Rectangular Waveguides and a Circular Waveguide

G. G. Gentili and A. Melloni

Abstract— **The paper describes a method for the analysis of the four-port junction (X-junction) between two rectangular waveguides and a circular waveguide. The method is based on the computation by an efficient technique of the generalized admittance matrix (GAM) of the structure (conveniently divided into subblocks) with no restrictions on the geometrical dimensions. a typical analysis with about 100 modes in the overall structures requires 1–4 s per frequency point on a typical medium-power workstation.**

I. Introduction

THE JUNCTION between one or two rectangular waveguides and a circular waveguides is a typical component of cylindrical cavity filters and of several different types of transducers, from rectangular to circular waveguide. In the literature, such a junction has been analyzed with the "small curvature" approximation [1]–[3], i.e., neglecting the curvature effect at the junction (see Fig. 1) and with a TE^x-field expansion [2], [3]. That approach was convenient since the purpose of those works was to develop a software tool for an efficient analysis TE_{011}-mode cylindrical cavity filters. A rigorous analysis which includes the curvature effects can be found in [4], where a mode-matching technique in conjunction with BCMM [5] was used to analyze the structure.

The method described in this letter is a considerable improvement over [1]–[3]: a complete TE/TM field expansion has been used (which is more general than the simpler TE^x expansion), and the small curvature approximation has been removed. With respect to [4], in this work the GAM approach has been used. Moreover, a different method for the analysis of the transition region has been employed, too. The result is a very powerful yet efficient tool for the analysis of the four-port structure shown in Fig. 1, with no limitations on the physical dimensions and on the range of frequency for which accurate results are expected. The results obtained in this work have been compared with those obtained by the well-known electromagnetic simulator HFSS™, by Hewlett Packard, and an excellent agreement has been observed. A remarkable efficiency of the method developed has been noticed and typical CPU times for the analysis of a four-port junction with 100 modes in the whole structure are about 1–4 s (depending on the CPU) per frequency point on a typical workstation.

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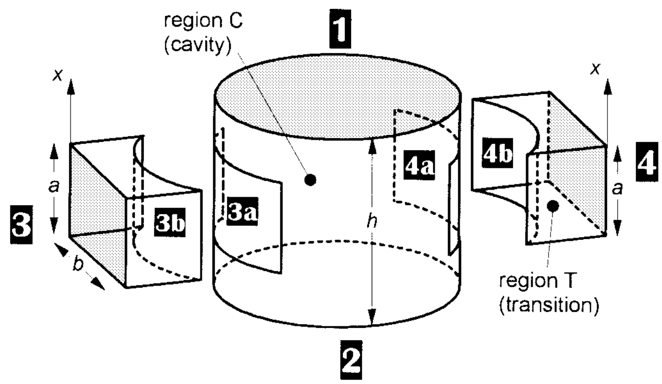

Fig. 1. Geometry of the X junction.

II. Formulation

The method is based on the computation of the Generalized Admittance Matrix (GAM) of the two building blocks shown in Fig. 1. The two blocks are the cavity (circular waveguide bounded by electric walls) and the region representing the transition from the rectangular waveguide to the curved surface in contact with the cavity. The two GAM's are then combined to form the GAM of the overall structure, from which the Generalized Scattering Matrix (GSM) can be readily computed. Since formulas for GAM combination and GAM to GSM transformation are well known, we concentrate our attention of the computation of the GAM for the two regions in Fig. 1.

A. GAM of the Cavity Region

Let $\mathbf{Y}^C$ be the GAM of the four-port cavity region shown in Fig. 1. Two ports are circular waveguide ports and two are lying on the curved surface. A GAM can be defined for that structure and it can be computed as follows: a set of normalized basis function $\{\mathbf{b}_n\}$ is defined for the expansion of the tangential electric field, a set of normalized testing functions $\{\mathbf{t}_n\}$ are defined for tangential magnetic field testing, and a dyadic operator $\overline{\mathbf{L}}$ is introduced yielding the tangential magnetic field on the boundary of the region generated by an assigned tangential electric field on the same boundary. $\overline{\mathbf{L}}$ can be expressed by a dyadic Green's function of the admittance type $\overline{\mathbf{G}}(\mathbf{r}, \mathbf{r}')$. Then, the generic element $\mathbf{Y}^C_{m,n}$ of matrix $\mathbf{Y}^C$

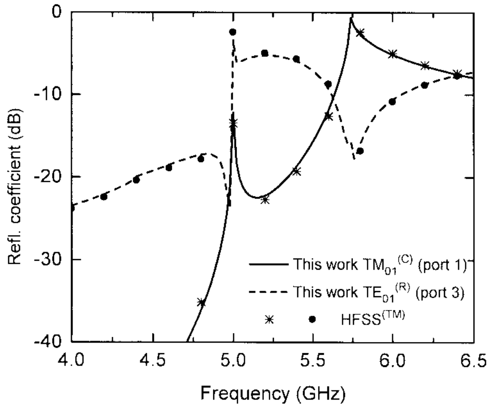

Fig. 2. Magnitude of reflection coefficient of mode TE_{01} at port 3 and mode TM_{01} at port 1. Comparison between this work and HFSS™.

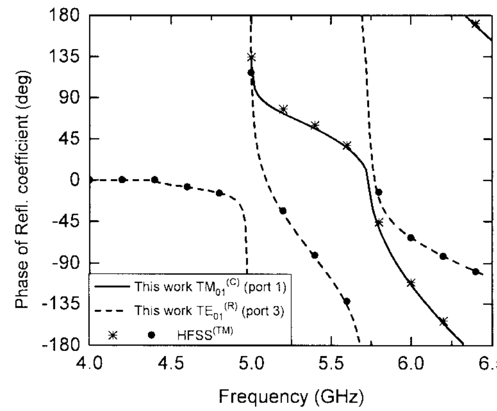

Fig. 3. Phase of reflection coefficient of mode TE_{01} at port 3 and mode TM_{01} at port 1 ($d' = 1$ cm). Comparison between this work and HFSS™.

can be expressed as

$$\mathbf{Y}^C_{m,n} = \int_S \mathbf{t}_m(\mathbf{r}) \cdot \int_S \overline{\mathbf{G}}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{b}_n(\mathbf{r}')\, dS'\, dS. \quad (1)$$

When the source (electric field) is located on the curved region (ports 3a and 4a in Fig. 2) a sine/cosine vector field expansion can be used and function $\overline{\mathbf{G}}(\mathbf{r}, \mathbf{r}')$ can be conveniently expressed by a radial waveguide mode expansion. When the source is located at ports 1 and 2 (circular waveguide), a TE/TM circular waveguide modes expansion for both the source and function $\overline{\mathbf{G}}(\mathbf{r}, \mathbf{r}')$ has been used. This is the most convenient choice, mainly because the resulting elements of matrix $\mathbf{Y}^C$ are obtained in closed form. In both cases, the testing functions are defined as $\{\mathbf{t}_n\} = \{\hat{\mathbf{n}} \times \mathbf{b}_n\}$ where $\hat{\mathbf{n}}$ is a unit vector normal to the boundary and pointing inward. This choice is convenient because of the resulting symmetry of matrix $\mathbf{Y}^C$.

B. GAM of the Transition Region

Let $\mathbf{Y}^T$ be the GAM of the four-port transition region shown in Fig. 1. $\mathbf{Y}^C$ is block-diagonal and takes on the form

$$\mathbf{Y}^T = \begin{bmatrix} \mathbf{Y}^{T3} & 0 \\ 0 & \mathbf{Y}^{T4} \end{bmatrix} \quad (2)$$

where the superscript 3 indicates the submatrix relative to ports 3–3b and superscript 4 that relative to ports 4–4b in Fig. 1. The generic submatrix can be conveniently computed by expressing the fields with two Hertzian-type potentials $\varphi \mathbf{u}_x$ and $\psi \mathbf{u}_x$.

The following boundary integral equations are then imposed on the two-dimensional (2-D) region transverse to x:

$$\int_B \varphi \frac{\partial W^\varphi}{\partial n} ds = \int_B W^\varphi \frac{\partial \varphi}{\partial n} ds \quad (3)$$

$$\int_B \psi \frac{\partial W^\psi}{\partial n} ds = \int_B W^\psi \frac{\partial \psi}{\partial n} ds \quad (4)$$

where W^φ and W^ψ are suitable testing functions. The two equations are then solved after expanding each potential and its normal derivative in a sum of sine/cosine-type basis functions (with respect to the curvilinear coordinate) with unknown coefficients. The efficiency and accuracy of the resulting algorithm is greatly dependent on the choice of the testing functions. The choice made in this work is particularly convenient and it consists in a suitable set of *rectangular* waveguide mode functions. This is a major difference with respect to [4]. The final GAM of the transition region is then found after simple but lengthy matrix algebra and is omitted. Owing to the wise choice of testing functions in (3), (4), the final algorithm is extremely efficient and matrix $\mathbf{Y}^T$ is obtained in fractions of a second.

III. Results

Several structures have been analyzed and the results have been compared with those obtained by the electromagnetic simulator HFSS™. Each analysis of the structure in Fig. 1 generates a large amount of data and it is not possible to provide a complete comparison between the two sets of results. As a reference, some results are presented for a structure for which $a = 10$ mm, $b = 30$ mm, $h = 60$ mm, $D = 40$ mm (diameter of the circular waveguide), and $d' = 10$ mm. The structure has been chosen with a large b/D ratio so that the small curvature approximation would not be applicable in this case.

Only two S-parameters are shown and the results are presented in Figs. 2 and 3. The parameters are reflection coefficients (magnitude and phase) of the four-port structure for modes TE_{01} in rectangular waveguide (dominant mode, cutoff at 4.99 GHz) and TM_{01} in circular waveguide (cutoff at 5.73 GHz). A good agreement between the two sets of data can be observed in Figs. 2 and 3. Setting $h = 60$ mm, a rather small number of modes can be used in this analysis (14 in each circular waveguide and ten in each rectangular waveguide) and the total CPU time is less than 1 s per frequency point on a HP 9000 Mod 715/75.

IV. Conclusions

A rigorous and efficient method has been developed for the analysis of the X-junction between two rectangular waveguides and one circular waveguide. The method is based on the computation of the GAM of two substructures, which are then combined together and converted to GSM. A new efficient method for GAM computation of the transition from the rectangular waveguide to the curved surface in contact with the circular waveguide has been developed.

The results obtained have been compared with HFSS™ and the accuracy of the method presented have been confirmed.

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